BIG Ideas

- Select and use appropriate operations to solve problems and justify solutions.
- Communicate mathematics through informal and mathematical language, representations, and models.

Key Vocabulary

- absolute value (p. 80)
- integers (p. 78)
- negative number (p. 78)
- opposites (p. 88)

Real-World Link

Golf The scoring system in golf is based on integers. A positive score is over par, a negative score is under par, and a score of 0 is par.

Operations with Integers Make this Foldable to help you organize your notes about operations with integers. Begin with a sheet of grid paper.

1. Fold in half.

2. Fold the top to the bottom twice.

3. Open and cut along the second fold to make four tabs.

4. Fold lengthwise. Draw a number line on the outside. Label each tab as shown.
Example 3

Use the coordinate plane to write the ordered pair that names point A.

Step 1 Start at the origin.

Step 2 Move right on the x-axis to find the x-coordinate of point A, which is 4.

Step 3 Move up the y-axis to find the y-coordinate, which is 1.

The ordered pair for point A is (4, 1).

Example 2

Find the next term in the list. 6, 11, 16, 21, …

<table>
<thead>
<tr>
<th>6</th>
<th>11</th>
<th>16</th>
<th>21</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

The next term is 21 + 5 or 26.

Example 1

Evaluate $a - 2b + 3c$ if $a = 1$, $b = 2$, and $c = 3$.

$a - 2b + 3c = 1 - 2(2) + 3(3)$

Replace $a$ with 1, $b$ with 2, and $c$ with 3.

$= 1 - 4 + 9$

Multiply.

$= 6$

Simplify.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

Evaluate each expression if $a = 4$, $b = 10$, and $c = 8$. (Lesson 1-3)

1. $a + b + c$
2. $bc - ab$
3. $b + ac$
4. $4c + 3b$

5. SALES Laura sold three times as many bottles of water on Sunday as on Saturday. How many bottles did she sell Saturday if she sold 120 bottles Sunday? (Lesson 1-3)

Find the next term in each list. (Lesson 1-1)

6. 34, 40, 46, 52, 58, …
7. 135, 120, 105, 90, 75, …
8. PHONE The telephone company charges $0.30 for the first minute and $0.15 for each additional minute. How much would it cost to talk for 10 minutes? (Lesson 1-1)

Use the coordinate plane to name each ordered pair. (Prerequisite Skill)

9. (1, 3)
10. (5, 2)
11. (5, 5)
12. (3, 4)
13. (0, 2)
14. (6, 1)
15. (6, 4)
16. (4, 2)
17. (1, 1)
18. (2, 5)
19. ANIMALS A mole can dig a tunnel 300 feet long in one night. (Prerequisite Skill)

19. Make a table of ordered pairs in which the x-coordinate represents the number of nights and the y-coordinate represents the tunnel length for 1, 2, 3, and 4 nights.

20. Graph the ordered pairs.
The western United States was unusually dry in 2002. In the graph, a value of \(-6\) represents 6 inches below the normal rainfall.

a. What does a value of \(-3\) represent?
b. Which city was farthest from its normal rainfall?
c. How could you represent 5 inches above normal rainfall?

**Compare and Order Integers** With normal rainfall as the starting point of 0, you can express 6 inches below normal as \(0 - 6\), or \(-6\). A **negative number** is a number less than zero.

Negative numbers like \(-6\), positive numbers like \(+6\), and zero are members of the set of **integers**. Integers can be represented as points on a number line.

This set of integers can be written \(\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}\), where \(\ldots\) means continues indefinitely.

**EXAMPLE** Write Integers for Real-World Situations

Write an integer for each situation.

a. 500 feet below sea level  
   The integer is \(-500\).

b. a temperature increase of 12°  
   The integer is \(+12\).

c. a loss of $240  
   The integer is \(-240\).

**CHECK Your Progress**

1A. a loss of 8 yards  
   1B. a deposit of $15
Reading Math

Inequality Symbols
Read the symbol < as is less than. Read the symbol > as is greater than.

Real-World Link
Annika Sorenstam won the 2004 LPGA Championship at 13 under par. She was the LPGA’s leading money winner from 2001 to 2004.
Source: LPGA.com

Lesson 2-1 Integers and Absolute Value

To graph integers, locate the points named by the integers on a number line. The number that corresponds to a point is called the coordinate of that point.

Notice that the numbers on a number line increase as you move from left to right. This can help you determine which of two numbers is greater.

Any mathematical sentence containing < or > is called an inequality. An inequality compares numbers or quantities.

EXAMPLE Compare Two Integers

Use the integers graphed on the number line below.

a. Write two inequalities involving -3 and 4.
   Since -3 is to the left of 4, write -3 < 4.
   Since 4 is to the right of -3, write 4 > -3.

b. Replace the • with <, > or = in -5 • -1 to make a true sentence.
   -1 is greater since it lies to the right of -5. So write -5 < -1.

2A. Write two inequalities involving -2 and -6.

2B. Replace the • with < or > in 2 • -1 to make a true sentence.

GOLF The top ten fourth-round scores of the 2004 LPGA Championship tournament were 0, +1, -5, -2, -1, +4, +2, +3, +5, and -3. Order the scores from least to greatest.

Real-World EXAMPLE

Graph each integer on a number line.

Write the numbers as they appear from left to right. The scores -5, -3, -2, -1, 0, +1, +2, +3, +4, and +5 are in order from least to greatest.

3. GOLF The top ten fourth-round scores of the 2004 PGA Championship were +4, -2, +6, +1, -4, -3, +5, -1, +2, and +3. Order the scores from least to greatest.

Source: LPGA.com
**Absolute Value** On the number line, notice that \(-5\) and 5 are each 5 units from 0, even though they are on opposite sides of 0. Numbers that are the same distance from zero have the same absolute value.

The symbol for absolute value is two vertical bars on either side of the number.

\[ |5| = 5 \quad \text{The absolute value of 5 is 5.} \]
\[ |-5| = 5 \quad \text{The absolute value of } -5 \text{ is 5.} \]

**KEY CONCEPT**

**Absolute Value**

**Words** The absolute value of a number is the distance the number is from zero on the number line. The absolute value of a number is always greater than or equal to zero.

**Examples**  
\[ |5| = 5 \quad | -5 | = 5 \]

**EXAMPLE** Expressions with Absolute Value

Evaluate each expression.

\[ | -8 | \]

-8 units

\[ |-8| = 8 \quad \text{The graph of } -8 \text{ is 8 units from 0.} \]

b. \[ |9| + | -7| \]

\[ |9| + | -7| = 9 + 7 \quad \text{The absolute value of } -7 \text{ is 7.} \]
\[ = 16 \quad \text{Simplify.} \]

**CHECK Your Progress**

4A. \[ | -3 | \]
4B. \[ -4 \] - \[ |3| \]

Since variables represent numbers, you can use absolute value notation with algebraic expressions involving variables.

**EXAMPLE** Algebraic Expressions with Absolute Value

**ALGEBRA** Evaluate \( |x| - 3 \) if \( x = -5 \).

\[ |x| - 3 = | -5 | - 3 \quad \text{Replace } x \text{ with } -5. \]
\[ = 5 - 3 \quad \text{The absolute value of } -5 \text{ is 5.} \]
\[ = 2 \quad \text{Simplify.} \]

5. Evaluate \( |y| + 8 \) if \( x = -7 \).
Write an integer for each situation. Then graph on a number line.

1. $8^\circ$ below zero
2. a 15-yard gain

Write two inequalities using the numbers in each sentence. Use the symbols < or >.

3. $-7^\circ$ is colder than $3^\circ$.
4. $-6$ is greater than $-10$.

Replace each $\oplus$ with $<$, $>$, or $=$ to make a true sentence.

5. $-18 \oplus -8$
6. $0 \oplus -3$
7. $9 \oplus -9$

8. Order the integers $\{28, -6, 0, -2, 5, -52, 115\}$ from least to greatest.

9. TEST DRIVES The table shows the recorded acceleration for a new car at regular intervals. Order the accelerations from least to greatest.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration (m/s²)</td>
<td>16</td>
<td>-31</td>
<td>-10</td>
<td>-6</td>
<td>9</td>
<td>34</td>
<td>-23</td>
<td>-19</td>
<td>-2</td>
</tr>
</tbody>
</table>

Evaluate each expression.

10. $| -10 |$
11. $|10| - |-4|$
12. $|16| + |-5|$

ALGEBRA Evaluate each expression if $a = -8$ and $b = 5$.

13. $9 + |a|$
14. $|a| - b$
15. $2|a|$

Write an integer for each situation. Then graph on a number line.

16. a bank withdrawal of $100
17. a loss of 6 pounds
18. a salary increase of $250
19. a gain of 9 yards
20. $12^\circ$ above zero
21. 5 seconds before liftoff

Write two inequalities using the numbers in each sentence. Use the symbols < or >.

22. 3 meters is taller than 2 meters.
23. A temperature of $-5^\circ$F is warmer than a temperature of $-10^\circ$F.
24. 55 miles per hour is slower than 65 miles per hour.
25. A 4-yard loss is less than no gain.

Replace each $\oplus$ with $<$, $>$, or $=$ to make a true sentence.

26. $-6 \oplus -2$
27. $-10 \oplus -13$
28. $0 \oplus -9$
29. $14 \oplus 0$
30. $-18 \oplus 8$
31. $5 \oplus -23$
32. $|9| \oplus |-9|$
33. $| -20 | \oplus |-4|$

Order the integers in each set from least to greatest.

34. $\{5, 0, -8\}$
35. $\{-15, -1, -2, -4\}$
36. $\{19, -16, 4, 62, -80\}$
37. $\{41, -14, 50, -23, -20\}$
38. $\{24, 5, -46, 9, 0, -3\}$
39. $\{98, -57, -60, 38, 188\}$

Lesson 2-1 Integers and Absolute Value 81
Evaluate each expression.

40. $|−15|$  
41. $|46|$  
42. $−|20|$  
43. $−|5|$  
44. $|0|$  
45. $|7|$  
46. $|−5| + |4|$  
47. $|0| + |−2|$  
48. $|15| − |−1|$  
49. $|0 + 9|$  
50. $|9 − 5| − |6 − 8|$  
51. $−|−6 + 1| − |5 − 6|$  

**ALGEBRA** Evaluate each expression if $a = 0$, $b = 3$, and $c = −4$.

52. $14 + |b|$  
53. $|c| − a$  
54. $a + b + |c|$  
55. $ab + |−40|$  
56. $|c| − b$  
57. $|ab| + b$

58. **GEOGRAPHY** The Caribbean Sea has an average depth of 8448 feet below sea level. Use an integer to express this depth.

59. Graph the temperatures on a number line.

60. Compare the lowest temperature in Alaska and the lowest temperature in Wisconsin using the $<$ symbol.

61. Compare the lowest temperature in Montana and the lowest temperature in Wisconsin using the $>$ symbol.

62. Write the temperatures in order from greatest to least.

63. $\{0, −2, 4\}$  
64. $\{−3, 1, 2, 5\}$  
65. $\{−2, −4, −5, −8\}$  
66. $\{−4, 0, 6, −7, −1\}$  

67. Name the coordinates of each point graphed on the number line.

68. **SOLAR SYSTEM** The average temperature of Saturn is $−218°F$ while the average temperature of Jupiter is $−162°F$. Which planet has the lower average temperature? Explain.

69. **FIND THE DATA** Refer to the United States Data File on pages 18–21. Choose some data and write a real-world problem in which you would compare and order integers.

70. **OPEN ENDED** Write two inequalities using integers.

71. **NUMBER SENSE** Explain how to find the number of units apart $−4$ and $5$ are on a number line.

72. **Which One Doesn’t Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.

\[
\begin{align*}
|12 − |−4|| & \quad |−2| + |4| & \quad −|7 + 1| & \quad |−8|
\end{align*}
\]
CHALLENGE  Consider two numbers $A$ and $B$ on a number line.
73. Is it always, sometimes, or never true that the distance between $A$ and $B$ equals the distance between $|A|$ and $|B|$? Explain.

75. **Writing in Math**  Use the information about integers on page 78 to explain how they can be used to model real-world situations. Include an explanation of how integers are used to describe rainfall.

76. **What is the temperature shown on the thermometer at the right?**
   A  8°F  
   B  7°F  
   C  −7°F  
   D  −8°F  

77. **GRIDDABLE**  How many units apart are −4 and 3 on a number line?

78. Which of the following describes the absolute value of −2°?
   F  It is the distance from −2 to 2 on the thermometer.
   G  It is the distance from −2 to 0 on the thermometer.
   H  It is the actual temperature outside when the thermometer reads −2°.
   J  None of these describes the absolute value of −2°.

Spiral Review

Determine whether a scatter plot of the data for the following might show a positive, negative, or no relationship. Explain your answer.  (Lesson 1-7)

79. height and arm length  
80. birth month and weight

Express each relation as a table and as a list of ordered pairs.  (Lesson 1-6)

81.  
82.  

Name the property shown by each statement.  (Lesson 1-4)

83.  $20(18) = 18(20)$  
84.  $9(8)(0) = 0$  
85.  $3ab = 3ba$

**PREREQUISITE SKILL**  Find each sum or difference.

86.  $18 + 29 + 46$  
87.  $232 + 156$  
88.  $451 + 629 + 1027$

89.  $36 - 19$  
90.  $479 - 281$  
91.  $2011 - 962$
Algebra Lab
Adding Integers

In a set of algebra tiles, $\text{1}$ represents the integer 1, and $\text{-1}$ represents the integer $-1$. You can use algebra tiles and an integer mat to model operations with integers.

**ACTIVITY 1**

The following example shows how to find the sum $-3 + (-2)$ using algebra tiles. Remember that addition means combining. The expression $-3 + (-2)$ tells you to combine a set of 3 negative tiles with a set of 2 negative tiles.

Therefore, $-3 + (-2) = -5$.

There are two important properties to keep in mind when you model operations with integers.
- When one positive tile is paired with one negative tile, the result is called a zero pair.
- You can add or remove zero pairs from a mat because removing or adding zero does not change the value of the tiles on the mat.

The following example shows how to find the sum $-4 + 3$.

Therefore, $-4 + 3 = -1$. 

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84 Chapter 2 Integers
**Exercises**

Use algebra tiles to model and find each sum.

1. \(-2 + (-4)\)  
2. \(-3 + (-5)\)  
3. \(-6 + (-1)\)  
4. \(-4 + (-5)\)  
5. \(-4 + 2\)  
6. \(2 + (-5)\)  
7. \(-1 + 6\)  
8. \(4 + (-4)\)

**Activity 2**

The Addition Table was completed using algebra tiles. In the highlighted portions of the table, the addends are \(-3\) and 1, and the sum is \(-2\). So, \(-3 + 1 = -2\). You can use the patterns in the Addition Table to learn more about integers.

<table>
<thead>
<tr>
<th>Addition Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+)</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
</tr>
<tr>
<td>-4</td>
</tr>
</tbody>
</table>

**Analyze the Results**

9. **MAKE A CONJECTURE** Locate all of the positive sums in the table. Describe the addends that result in a positive sum.

10. **MAKE A CONJECTURE** Locate all of the negative sums in the table. Describe the addends that result in a negative sum.

11. **MAKE A CONJECTURE** Locate all of the sums that are zero. Describe the addends that result in a sum of zero.

12. The Identity Property says that when zero is added to any number, the sum is the number. Does it appear that this property is true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.

13. The Commutative Property says that the order in which numbers are added does not change the sum. Does it appear that this property is true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.

14. The Associative Property says that the way numbers are grouped when added does not change the sum. Is this property true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.


**Main Ideas**
- Add two integers.
- Add more than two integers.

**New Vocabulary**
- opposites
- additive inverse

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In football, forward progress is represented by a positive integer. Being pushed back is represented by a negative integer. On the first play a team loses 5 yards and on the second play they lose 2 yards.

**a.** What integer represents the total yardage on the two plays?

**b.** Write an addition sentence that describes this situation.

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**Add Integers** The equation \(-5 + (-2) = -7\) is an example of adding two integers with the same sign. Notice that the sign of the sum is the same as the sign of the addends.

Recall that the numbers you add are called *addends*. The result is called the *sum*.

---

**EXAMPLE**

**Add Integers on a Number Line**

1. Find \(-2 + (-3)\)

   ![Number Line Diagram]

   Start at zero. Move 2 units to the left. From there, move 3 more units to the left. 

   \(-2 + (-3) = -5\)

   **CHECK Your Progress**

   **1A.** \(-3 + (-4)\)  **1B.** \(-6 + (-14)\)

   This example suggests a rule for adding integers with the same sign.

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**KEY CONCEPT**

**Adding Integers with the Same Sign**

**Words**
To add integers with the same sign, add their absolute values. The sum is:
- positive if both integers are positive.
- negative if both integers are negative.

**Examples** \(-5 + (-2) = -7\) \(6 + 3 = 9\)
EXAMPLE

Add Integers with the Same Sign

Find \(-4 + (-5)\).
\[-4 + (-5) = -9\] Add \(|-4| \text{ and } |-5|\). The sum is negative.

Find each sum.

\[\text{2A. } -8 + (-2) \quad \text{2B. } -1 + (-12)\]

A number line can also help you understand how to add integers with different signs.

EXAMPLE

Add Integers on a Number Line

Find each sum.

\[\text{a. } 7 + (-4) \quad \text{b. } 2 + (-3)\]

\[\begin{align*}
\text{Start at zero. Move 7 units to the right.} \\
\text{From there, move 4 units to the left.} \\
\text{7} + (-4) = 3 \\
\end{align*}\]

\[\begin{align*}
\text{Start at zero. Move 2 units to the right.} \\
\text{From there, move 3 units to the left.} \\
2 + (-3) = -1 \\
\end{align*}\]

\[\text{3A. } 5 + (-2) \quad \text{3B. } 4 + (-8)\]

Notice how the sums in Example 3 relate to the addends. The sign of the sum is the same as the sign of the addend with the greater absolute value.

KEY CONCEPT

Adding Integers with Different Signs

To add integers with different signs, subtract their absolute values. The sum is:
- positive if the positive integer’s absolute value is greater.
- negative if the negative integer’s absolute value is greater.

EXAMPLE

Add Integers with Different Signs

Find each sum.

\[\text{a. } -8 + 3 \quad \text{b. } 10 + (-4)\]

\[\begin{align*}
-8 + 3 &= -5 \\
10 + (-4) &= 6 \\
\end{align*}\]

To find \(-8 + 3\), subtract \(|3|\) from \(|-8|\).
The sum is negative because \(|-8| > |3|\).

To find \(10 + |-4|\), subtract \(|-4|\) from \(|10|\).
The sum is positive because \(|10| > |-4|\).

\[\text{4A. } -9 + 4 \quad \text{4B. } 12 + (-5)\]
**Real-World Link**
The temperatures on the moon are so extreme because the moon does not have any atmosphere to trap heat.

**ASTRONOMY** During the night, the average temperature on the moon is $-140^\circ\text{C}$. By noon, the average temperature has risen $252^\circ\text{C}$. What is the average temperature on the moon at noon?

<table>
<thead>
<tr>
<th>Words</th>
<th>Temperature at night plus increase by noon equals temperature at noon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Let $x$ = the temperature at noon.</td>
</tr>
<tr>
<td>Equation</td>
<td>$-140 + 252 = x$</td>
</tr>
</tbody>
</table>

Solve the equation. **Estimate** $-140 + 250 = 110$.

$-140 + 252 = x$  To find the sum, subtract $| -140 |$ from $252$.

$112 = x$  The sum is positive because $| 252 | > | -140 |$.

The average temperature at noon is $112^\circ\text{C}$. The solution is reasonable to the estimate.

**CHECK Your Progress**

5. **SUBMARINES** A submarine was at a depth of 103 feet below the surface of the water. It rose 68 feet. What is its current depth?

**Add More Than Two Integers** Two numbers with the same absolute value but different signs are called **opposites**. For example, $-4$ and $4$ are opposites. An integer and its opposite are also called **additive inverses**.

**KEY CONCEPT**

**Additive Inverse Property**

<table>
<thead>
<tr>
<th>Words</th>
<th>The sum of any number and its additive inverse is zero.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>$x + (-x) = 0$</td>
</tr>
<tr>
<td>Example</td>
<td>$6 + (-6) = 0$</td>
</tr>
</tbody>
</table>

**EXAMPLE** Find each sum.

6. **Add Three or More Integers**

   a. $9 + (-3) + (-9)$

   $9 + (-3) + (-9) = 9 + (-9) + (-3)$  Commutative Property

   $= 0 + (-3)$  Additive Inverse Property: $9 + (-9) = 0$

   $= -3$  Identity Property of Addition

   b. $-4 + 6 + (-3) + 9$

   $-4 + 6 + (-3) + 9 = -4 + (-3) + 6 + 9$  Commutative Property

   $= [(-4 + (-3))] + (6 + 9)$  Associative Property

   $= -7 + 15$  Simplify

   $= 8$

**6A.** $4 + (-2) + (-7)$  **6B.** $-10 + 3 + (-7) + 12$
Find each sum.

1. \(-2 + (-4)\)  
2. \(-10 + (-5)\)  
3. \(-14 + (-4)\)  
4. \(7 + (-2)\)  
5. \(11 + (-3)\)  
6. \(8 + (-5)\)  
7. \(2 + (-16)\)  
8. \(9 + (-12)\)  
9. \(-15 + 4\)

Example 5  
(p. 88)

10. **FOOTBALL** A team gained 4 yards on one play. On the next play, they lost 5 yards. Write an addition sentence to find the total yardage.

Example 6  
(p. 88)

Find each sum.

11. \(8 + (-6) + 2\)  
12. \(-6 + 5 + (-10)\)

Write an addition sentence for each situation. Then find the sum.

31. **GAME SHOWS** A contestant has \(-1500\) points. Suppose he loses another 1250 points.

32. **STOCKS** A stock price increases \$6. It then decreases \$10.

Find each sum.

13. \(-4 + (-1)\)  
14. \(-5 + (-2)\)  
15. \(-4 + (-6)\)  
16. \(-3 + (-8)\)  
17. \(-7 + (-8)\)  
18. \(-12 + (-4)\)  
19. \(-9 + (-14)\)  
20. \(-15 + (-6)\)  
21. \(-11 + (-15)\)  
22. \(-23 + (-43)\)  
23. \(8 + (-5)\)  
24. \(6 + (-4)\)  
25. \(3 + (-7)\)  
26. \(4 + (-6)\)  
27. \(-15 + 6\)  
28. \(-5 + 11\)  
29. \(18 + (-32)\)  
30. \(-45 + 19\)

41. **MONEY** The starting balance in a checking account was \$50. What was the balance after checks were written for \$25 and for \$32? Use estimation to determine whether your answer is reasonable.

42. **HIKING** Sally starts hiking at an elevation of 324 feet. She descends to an elevation of 201 feet and then ascends to an elevation 55 feet higher than where she began. She descends 183 feet. Describe the overall change in elevation.

Find each sum.

33. \(6 + (-9) + 9\)  
34. \(7 + (-13) + 4\)  
35. \(-9 + 16 + (-10)\)  
36. \(-12 + 18 + (-12)\)  
37. \(14 + (-9) + 6\)  
38. \(28 + (-35) + 4\)  
39. \(-41 + 25 + (-10)\)  
40. \(-18 + 35 + (-17)\)

43. \(|18 + (-13)|\)  
44. \(|-27 + 19|\)  
45. \(|-25 + (-12)|\)
**POPULATION** For Exercises 46 and 47, use the table that shows the change in population of several cities from 2002 to 2003.

<table>
<thead>
<tr>
<th>City</th>
<th>2002 Population</th>
<th>Change as of 2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Paso, TX</td>
<td>577,415</td>
<td>6698</td>
</tr>
<tr>
<td>San Jose, CA</td>
<td>900,443</td>
<td>−2094</td>
</tr>
<tr>
<td>Lexington, KY</td>
<td>263,618</td>
<td>3180</td>
</tr>
<tr>
<td>Columbia, SC</td>
<td>117,394</td>
<td>−37</td>
</tr>
</tbody>
</table>

Source: The World Almanac

46. What was the population in each city in 2003?
47. What was the total change in population of these cities?

**H.O.T. Problems:**

48. **OPEN ENDED** Give an example of two integers that are additive inverses.

49. **CHALLENGE** True or false? $-n$ always names a negative number. If false, give a counterexample.

**CHALLENGE** Name the property illustrated by each of the following.

50. $a(b + (-b)) = (b + (-b))a$
51. $a(b + (-b)) = 0$
52. **Writing in Math** Explain how a number line can help you add integers.

53. A Guadelupe bass was swimming underwater at a depth of 12 feet. It rose 3 feet, dropped 5 feet, rose 10 feet, and dropped 1 foot. What is the current depth of the fish?
   A −7 ft
   B −5 ft
   C −3 ft
   D 7 ft

54. Which expression is represented by the model?
   F $-5 + -1$
   H $-5 + 1$
   G $-5 + 0$
   J $-5 + 4$

55. **CHEMISTRY** The freezing point of oxygen is 219 degrees below zero on the Celsius scale. Use an integer to express this temperature. (Lesson 2-1)

Determine whether a scatter plot of the data for the following might show a positive, negative, or no relationship. (Lesson 1-7)

56. age and family size
57. temperature and sales of mittens

**ALGEBRA** Find the solution of each equation from the list given. (Lesson 1-5)

58. $18 - n = 12; 3, 6, 30$
59. $7a = 49; 7, 42, 343$

**PREREQUISITE SKILL** Evaluate each expression if $a = 6, b = 10,$ and $c = 3$. (Lesson 1-3)

60. $a + 19$
61. $2b - 6$
62. $ab - ac$
63. $3a - (b + c)$
Learning Mathematics Vocabulary

Some words used in mathematics are also used in English and have similar meanings. For example, in mathematics add means to combine. The meaning in English is to join or unite.

Some words are used only in mathematics. For example, addend means a number to be added to another.

Some words have more than one mathematical meaning. For example, an inverse operation undoes the effect of another operation, and an additive inverse is a number that when added to a given number gives zero.

The list below shows some of the mathematics vocabulary used in Chapters 1 and 2.

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Meaning</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebraic</td>
<td>expression that contains at least one variable and at least one</td>
<td>$2 + x, \frac{4}{c}$, $3b$</td>
</tr>
<tr>
<td>expression</td>
<td>mathematical operation</td>
<td></td>
</tr>
<tr>
<td>evaluate</td>
<td>to find the value of an expression</td>
<td>$2 + 5 = 7$</td>
</tr>
<tr>
<td>simplify</td>
<td>to find a simpler form of an expression</td>
<td>$3b + 2b = 5b$</td>
</tr>
<tr>
<td>integer</td>
<td>a whole number, its additive inverse, or zero</td>
<td>$-3, 0, 2$</td>
</tr>
<tr>
<td>factor</td>
<td>a number that is multiplied by another number</td>
<td>$3(4) = 12$</td>
</tr>
<tr>
<td>product</td>
<td>the result of multiplying</td>
<td>$3(4) = 12 \leftarrow \text{product}$</td>
</tr>
<tr>
<td>quotient</td>
<td>the result of dividing two numbers</td>
<td>$\frac{12}{4} = 3 \leftarrow \text{quotient}$</td>
</tr>
<tr>
<td>dividend</td>
<td>the number being divided</td>
<td>$\frac{12}{4} = 3 \leftarrow \text{dividend}$</td>
</tr>
<tr>
<td>divisor</td>
<td>the number being divided into another number</td>
<td>$\frac{12}{4} = 3 \leftarrow \text{divisor}$</td>
</tr>
<tr>
<td>coordinate</td>
<td>a number that locates a point</td>
<td>$(5, 2)$</td>
</tr>
</tbody>
</table>

Reading to Learn

1. Name two of the words above that are also used in everyday English. Use the Internet, a dictionary, or another reference to find their everyday definition. How do the everyday definitions relate to the mathematical definitions?

2. Name two words above that are used only in mathematics.

3. Name two words above that have more than one mathematical meaning. List their meanings.
You can also use algebra tiles to model subtraction of integers. Remember one meaning of subtraction is to *take away*.

### Activity
Use algebra tiles to find each difference.

**a.** $7 - 4$
Place 7 positive tiles on the mat. Remove 4 positive tiles.

![Algebra Tiles](image1.png)

So, $7 - 4 = 3$

**b.** $-8 - (-3)$
Place 8 negative tiles on the mat. Remove 3 negative tiles.

![Algebra Tiles](image2.png)

So, $-8 - (-3) = -5$

**c.** $5 - (-2)$
Place 5 positive tiles on the mat and then remove 2 negative tiles. However, there are 0 negative tiles. First you must add 2 zero pairs to the set.

![Algebra Tiles](image3.png)

Then remove the 2 negative tiles.

![Algebra Tiles](image4.png)

So, $5 - (-2) = 7$.

**d.** $-6 - 2$
Place 6 negative tiles on the mat. Remove 2 positive tiles. Since there are no positive tiles, add 2 zero pairs to the mat.

![Algebra Tiles](image5.png)

Then remove the 2 positive tiles.

![Algebra Tiles](image6.png)

So, $-6 - 2 = -8$.

### Analyze the Results
Apply what you learned to find each difference.

1. $9 - 7$
2. $5 - (-3)$
3. $6 - (-3)$
4. $1 - (-5)$
5. $3 - (-9)$
6. $-8 - 3$
7. $-8 - (-1)$
8. $-1 - 4$

9. **Make a Conjecture** Write a rule that will help you determine the sign of the difference of two integers.
You can use a number line to subtract integers. The model below shows how to find $6 - 8$.

**Step 1** Start at 0. Move 6 units right to show positive 6.

**Step 2** From there, move 8 units left to subtract positive 8.

a. What is $6 - 8$?

b. What direction do you move to indicate subtracting a positive integer?

c. What addition sentence is also modeled by the number line above?

**Subtract Integers** When you subtract 8, as shown on the number line above, the result is the same as adding $-8$. When you subtract 5, the result is the same as adding $-5$. These examples suggest a method for subtracting integers.

$$6 - 8 = -2 \quad 6 + (-8) = -2 \quad -3 - 5 = -8 \quad -3 + (-5) = -8$$

**KEY CONCEPT**

**Subtracting Integers**

**Words** To subtract an integer, add its additive inverse.

**Symbols** $a - b = a + (-b)$

### Example

**Subtract a Positive Integer**

Find each difference.

a. $8 - 13$

$$8 - 13 = 8 + (-13) \quad \text{To subtract 13, add } -13.$$  
$$= -5 \quad \text{Simplify.}$$

b. $-4 - 10$

$$-4 - 10 = -4 + (-10) \quad \text{To subtract 10, add } -10.$$  
$$= -14 \quad \text{Simplify.}$$

**CHECK Your Progress**

1A. $9 - 16$  
1B. $-5 - 11$
In Example 1, you subtracted a positive integer by adding its additive inverse. Use inductive reasoning to see if the method also applies to subtracting a negative integer.

<table>
<thead>
<tr>
<th>Subtracting an Integer</th>
<th>Adding Its Additive Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 - 2 = 0$</td>
<td>$2 + (-2) = 0$</td>
</tr>
<tr>
<td>$2 - 1 = 1$</td>
<td>$2 + (-1) = 1$</td>
</tr>
<tr>
<td>$2 - 0 = 2$</td>
<td>$2 + 0 = 2$</td>
</tr>
<tr>
<td>$2 - (-1) = ?$</td>
<td>$2 + 1 = 3$</td>
</tr>
</tbody>
</table>

Continuing the pattern in the first column, $2 - (-1) = 3$. The result is the same as when you add the additive inverse.

**EXAMPLE** Subtract a Negative Integer

Find each difference.

a. $7 - (-3)$

$7 - (-3) = 7 + 3$

To subtract $-3$, add 3.

$= 10$

b. $-2 - (-4)$

$-2 - (-4) = -2 + 4$

To subtract $-4$, add 4.

$= 2$

**WEATHER** The table shows the record high and low temperatures in selected states as of a recent year. What is the range, or difference between the highest and lowest temperatures, for Virginia?

**Explore** You know the highest and lowest temperatures. You need to find the range for Virginia’s temperatures.

**Plan** To find the range, or difference, subtract the lowest temperature from the highest temperature.

**Solve** $110 - (-30) = 110 + 30$

To subtract $-30$, add 30.

$= 140$

Add 110 and 30.

The range for Virginia is $140^\circ$.

**Check** Think of a thermometer. The difference between $110^\circ$ above zero and $30^\circ$ below zero must be $110 + 30$ or $140^\circ$. The answer appears to be reasonable.

**3. WEATHER** What is the range of temperatures for Washington?
Evaluate Expressions You can use the rule for subtracting integers to evaluate expressions.

**EXAMPLE 4** Evaluate Algebraic Expressions

**a.** Evaluate \( x - (-6) \) if \( x = 12 \).

\[
x - (-6) = 12 - (-6) \quad \text{Write the expression. Replace } x \text{ with 12.}
\]

\[
= 12 + 6 \quad \text{To subtract } -6, \text{ add its additive inverse, 6.}
\]

\[
= 18 \quad \text{Add 12 and 6.}
\]

**b.** Evaluate \( a - b + c \) if \( a = 15, b = 5, \) and \( c = -8 \).

\[
a - b + c = 15 - 5 + (-8) \quad \text{Replace } a \text{ with 15, } b \text{ with 5, and } c \text{ with } -8.
\]

\[
= 10 + (-8) \quad \text{Order of operations}
\]

\[
= 2 \quad \text{Add 10 and } -8.
\]

Evaluate each expression if \( \ell = 7, m = -3, \) and \( n = -10 \).

**4A.** \( n - \ell \)

**4B.** \( \ell - m + n \)

**Examples 1, 2** (pp. 93–94)

Find each difference.

1. \( 8 - 11 \)
2. \( 10 - 15 \)
3. \( -10 - 14 \)
4. \( -9 - 3 \)
5. \( 7 - (-10) \)
6. \( 16 - (-12) \)
7. \( -6 - (-4) \)
8. \( -2 - (-8) \)
9. \( -15 - (-18) \)

**Example 3** (p. 94)

**ANIMALS** A gopher begins at 7 inches below the surface of a garden and digs down another 9 inches. Find an integer that represents the gopher’s position in relation to the surface of the garden.

**Example 4** (p. 95)

**ALGEBRA** Evaluate each expression if \( x = 10, y = -4, \) and \( z = -15 \).

11. \( x - (-10) \)
12. \( y - x \)
13. \( x + y - z \)

Find each difference.

14. \( 3 - 8 \)
15. \( 4 - 5 \)
16. \( 2 - 9 \)
17. \( 9 - 12 \)
18. \( -3 - 1 \)
19. \( -5 - 4 \)
20. \( -6 - 7 \)
21. \( -4 - 8 \)
22. \( 6 - (-8) \)
23. \( 4 - (-6) \)
24. \( 7 - (-4) \)
25. \( 9 - (-3) \)
26. \( -9 - (-7) \)
27. \( -7 - (-10) \)
28. \( -11 - (-12) \)
29. \( -16 - (-7) \)
30. \( 10 - 24 \)
31. \( 48 - (-50) \)

32. **MONEY** Suppose you deposited $25 into your checking account and wrote a check for $38. What was the change in your account balance?

33. **GEOGRAPHY** The highest point in California is Mt. Whitney, with an elevation of 14,494 feet. The lowest point is Death Valley, with an elevation of -282 feet. Find the difference in the elevations.
**REAL-WORLD LINK**

Consumers spent a total of $38.4 billion on their lawns and gardens in 2003.

Source: The National Gardening Association

**EXTRA PRACTICE**

See pages 764, 795.

Math Online

Self-Check Quiz at pre-alg.com

**H.O.T. Problems:**

**ALGEBRA** Evaluate each expression if \( x = -3 \), \( y = 8 \), and \( z = -12 \).

34. \( y - 10 \)  
35. \( 12 - z \)  
36. \( 3 - x \)  
37. \( z - 24 \)

38. \( x - y \)  
39. \( z - x \)  
40. \( y - z \)  
41. \( z - y \)

42. \( x + y - z \)  
43. \( z - y + x \)  
44. \( x - y - z \)  
45. \( z - y - x \)

**ANALYZE TABLES** For Exercises 46 and 47, use the table.

46. Describe the change in the sales related to each gardening activity from 2002 to 2003.

47. What was the total change in sales related to these gardening activities from 2002 to 2003?

**BUSINESS** The formula \( P = I - E \) relates profit \( P \) to income \( I \) and expenses \( E \). One month a small business has income of $19,592 and expenses of $20,345.

48. What is the profit for the month?

49. What does a negative profit mean?

Find each difference.

50. \( 125 - (-114) \)  
51. \( -320 - (-106) \)  
52. \( -2200 - (-3500) \)

**ANALYZE TABLES** The daily closing prices for a company’s stock during one week are shown in the table.

<table>
<thead>
<tr>
<th>Date</th>
<th>Nov. 3</th>
<th>Nov. 4</th>
<th>Nov. 5</th>
<th>Nov. 6</th>
<th>Nov. 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing Price</td>
<td>$33.30</td>
<td>$30.59</td>
<td>$31.04</td>
<td>$31.97</td>
<td>$30.15</td>
</tr>
</tbody>
</table>

53. Find the change in the closing price since the previous day.

54. What is the difference between the highest and lowest changes in closing price?

55. **FIND THE DATA** Refer to the United States Data File on pages 18–21. Choose some data and write a real-world problem in which you would subtract integers.

**CHALLENGE** Determine whether each statement is true or false. If false, give a counterexample.

56. Subtraction of integers is commutative.

57. Subtraction of integers is associative.

58. **FIND THE ERROR** José and Amy are finding \( 8 - (-2) \). Who is correct? Explain your reasoning.

José  
\[
8 - (-2) = 8 + 2 = 10
\]

Amy  
\[
8 - (-2) = 8 + 2 = 10
\]

59. **OPEN ENDED** Write examples of a positive and a negative integer and their additive inverses.
60. **SELECT A TECHNIQUE** Reiko is filling out her check register. Which technique(s) might Reiko use to find out if she spent more money than she had in her checking account? Justify your selection(s). Then use the technique(s) to find how much she has left in her account.

<table>
<thead>
<tr>
<th>Check No.</th>
<th>Date</th>
<th>Description</th>
<th>Payment</th>
<th>Deposit</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/4</td>
<td></td>
<td>Paycheck</td>
<td>$700</td>
<td></td>
<td>$700</td>
</tr>
<tr>
<td>104</td>
<td>2/6</td>
<td>School books</td>
<td>$275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>2/7</td>
<td>Initiation fee</td>
<td>$325</td>
<td></td>
<td></td>
</tr>
<tr>
<td>106</td>
<td>2/10</td>
<td>Graphing calculator</td>
<td>$150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

61. **Writing in Math** Use the information about subtracting integers on page 93 to explain how the addition and subtraction of integers are related.

62. The melting point of metal mercury is $-39^\circ C$. The freezing point of alcohol is $-114^\circ C$. How much warmer is the melting point of mercury than the freezing point of alcohol?
   A. $-153^\circ C$  
   B. $-75^\circ C$  
   C. $75^\circ C$  
   D. $153^\circ C$

63. The terms in a pattern are given in the table. What is the value of the fifth term?

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
</tbody>
</table>

64. **OCEANOGRAPHY** A submarine at 1300 meters below sea level descends an additional 1150 meters. What integer represents the submarine’s position with respect to sea level? *(Lesson 2-2)*

65. **ALGEBRA** Evaluate $|b| - |a|$ if $a = 2$ and $b = -4$. *(Lesson 2-1)*

**ALGEBRA** Translate each phrase into an algebraic expression. *(Lesson 1-3)*

66. A number divided by 5
67. The sum of $t$ and 9
68. The quotient of eighty-six and $b$
69. $s$ decreased by 8

Find the value of each expression. *(Lesson 1-2)*

60. $2 \times (5 + 8) - 6$
61. $96 \div (6 \times 8) \div 2$
62. $17 - (21 + 13) \div 17$

**PREREQUISITE SKILL** Find each product.

73. $5 \cdot 15$
74. $8 \cdot 12$
75. $3 \cdot 5 \cdot 8$
76. $4 \cdot 9 \cdot 12$
1. **MULTIPLE CHOICE** Choose the integer between 2 and −1. (Lesson 2-1)
   A −3  B −0.5  C 1  D 2.5

Replace each • with <, >, or = to make a true sentence. (Lesson 2-1)
2. 9 • −5
3. −3 • 0
4. −8 • −6
5. 2 • −4

6. **MULTIPLE CHOICE** Refer to the number line. Which statement is true? (Lesson 2-1)

   ![Number Line]

   F |B| < |C|  H B > C
   G C > A  J |D| > |A|

7. **TEMPERATURE** Order the temperatures from least to greatest. (Lesson 2-1)

<table>
<thead>
<tr>
<th>Item</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid helium</td>
<td>−452</td>
</tr>
<tr>
<td>Outer space</td>
<td>−457</td>
</tr>
<tr>
<td>Dry ice</td>
<td>−108</td>
</tr>
</tbody>
</table>

   Source: The Sizeaurus

Find each sum. (Lesson 2-2)
8. −5 + (−15)
9. −5 + 11
10. −6 + 9 + (−8)
11. 12 + (−6) + (−15)
12. |−33 + 19|

13. |−23 + −20|

14. **MULTIPLE CHOICE** Which day had the greatest change in stock price? (Lesson 2-2)

<table>
<thead>
<tr>
<th>Day</th>
<th>Open Price</th>
<th>Close Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon.</td>
<td>$43.29</td>
<td>$48.55</td>
</tr>
<tr>
<td>Tues.</td>
<td>$48.55</td>
<td>$46.65</td>
</tr>
<tr>
<td>Wed.</td>
<td>$46.65</td>
<td>$41.30</td>
</tr>
<tr>
<td>Thurs.</td>
<td>$41.30</td>
<td>$45.99</td>
</tr>
</tbody>
</table>


15. **SPACE** During night, the average temperature on Mars is −140°F. During the day, the average temperature rises 208°F. What is the average daytime temperature on Mars? (Lesson 2-2)

16. **ACCOUNTING** A small company had the following profits and losses for a six-month period. How much did the company earn during this time period? (Lesson 2-2)

<table>
<thead>
<tr>
<th>Month</th>
<th>Profit/Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>−$3674</td>
</tr>
<tr>
<td>Feb.</td>
<td>$4013</td>
</tr>
<tr>
<td>Mar.</td>
<td>−$1729</td>
</tr>
<tr>
<td>Apr.</td>
<td>−$1415</td>
</tr>
<tr>
<td>May</td>
<td>$1808</td>
</tr>
<tr>
<td>Jun.</td>
<td>−$547</td>
</tr>
</tbody>
</table>

Find each difference. (Lesson 2-3)
17. 16 − 23
18. −15 − 8
19. 25 − (−7)
20. −16 − (−11)

**ALGEBRA** Evaluate each expression if \(x = 5\), \(y = −2\), and \(z = −3\). (Lesson 2-3)
21. \(x - y\)
22. \(z - 6\)
23. \(x - y - z\)

24. **WEATHER** If the temperature is −9°F and it drops 5°F overnight, what is the new temperature? (Lesson 2-3)

25. **ASTRONOMY** The graph shows the highest and lowest points of three planets. (Lesson 2-3)

   ![Graph of Mountains and Valleys]

   What is the range of each of the planets? Which planet has the greatest range?
**Algebra Lab**

**Multiplying Integers**

You can also use algebra tiles to model multiplication of integers. Remember that $2 \times 3$ means *two sets of three items*. So, you can show $2 \times 3$ by placing 2 sets of 3 positive tiles on a mat.

Similarly, you can model $2 \times (-3)$ by placing 2 sets of 3 negative tiles on the mat, as shown at the right.

If the first factor is negative, you will need to *remove* tiles from the mat.

**ACTIVITY**

**Step 1** The expression $-2 \times (-3)$ means to *remove* 2 sets of 3 negative tiles. To do this, first place $2 \times 3$ or 6 zero pairs on the mat.

**Step 2** Then remove 2 sets of 3 negative tiles from the mat. There are 6 positive tiles remaining. So, $-2 \times (-3) = 6$.

**EXERCISES**

Use algebra tiles to model and find each product.

1. $6 \times (-2)$  
2. $3 \times (-5)$  
3. $3 \times (-4)$  
4. $1 \times (-8)$  
5. $-4 \times (-2)$  
6. $-5 \times (-2)$  
7. $-7 \times (-1)$  
8. $-2 \times (-2)$  

9. Explain the meaning of $-2 \times 3$. Then find the product using algebra tiles.

Use algebra tiles to model and find each product.

10. $-4 \times 2$  
11. $-3 \times 5$  
12. $-2 \times 6$  
13. $-1 \times 3$

**ANALYZE THE RESULTS**

14. How are the operations $-3 \times 4$ and $4 \times (-3)$ the same? How do they differ?

15. **MAKE A CONJECTURE** Find a rule you can use to find the sign of the product of two integers given the sign of both factors.
Main Ideas
- Multiply integers.
- Simplify algebraic expressions.

The temperature drops 7°C for each 1 kilometer increase in altitude. A drop of 7°C is represented by \(-7\). So, the temperature change equals the altitude times \(-7\).

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Altitude \times \text{Rate of Change}</th>
<th>Temperature Change (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1((-7))</td>
<td>(-7)</td>
</tr>
<tr>
<td>2</td>
<td>2((-7))</td>
<td>(-14)</td>
</tr>
<tr>
<td>3</td>
<td>3((-7))</td>
<td>(-21)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>11</td>
<td>11((-7))</td>
<td>(-77)</td>
</tr>
</tbody>
</table>

a. Suppose the altitude is 4 kilometers. Write an expression to find the temperature change.

b. Use the pattern in the table to find \(4(\(-7\))\).

**Multiply Integers** Multiplication is repeated addition. So, \(3(\(-7\))\) means that \(-7\) is used as an addend 3 times.

\[
3(\(-7\)) = (\(-7\)) + (\(-7\)) + (\(-7\)) = -21
\]

By the Commutative Property of Multiplication, \(3(\(-7\)) = -7(3)\). This example suggests the following rule.

**KEY CONCEPT**

**Multiplying Integers with Different Signs**

**Words** The product of two integers with different signs is negative.

**Examples**

\[
4(\(-3\)) = -12 \\
-3(4) = -12
\]

**EXAMPLE** Multiply Integers with Different Signs

Find each product.

a. \(5(\(-6\))\)

\[
5(\(-6\)) = -30 \quad \text{The factors have different signs. The product is negative.}
\]

b. \(\(-4(16)\)\)

\[
\(-4(16) = -64 \quad \text{The factors have different signs. The product is negative.}
\]

1A. \(7(\(-8\))\)  
1B. \(\(-6(12)\)\)
The product of two positive integers is positive. What is the sign of the product of two negative integers? Use a pattern to find a rule.

One positive and one negative factor: Negative product

\[ (-4)(2) = -8 \]
\[ (-4)(1) = -4 \]
\[ (-4)(0) = 0 \]

Each product is 4 more than the previous product.

Two negative factors: Positive product

\[ (-4)(-1) = 4 \]
\[ (-4)(-2) = 8 \]

\[ (-4)(-3) = 12 \]

\[ 4(-3) = 12 \]

Multiplying Integers with the Same Sign

<table>
<thead>
<tr>
<th>Words</th>
<th>The product of two integers with the same sign is positive.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>[ 4(3) = 12 ] [ -4(-3) = 12 ]</td>
</tr>
</tbody>
</table>
**Algebraic Expressions** You can use the rules for multiplying integers to simplify and evaluate algebraic expressions.

**EXAMPLE** Simplify and Evaluate Algebraic Expressions

a. Simplify \(-2x(3y)\).

\[-2x(3y) = (-2)(x)(3)(y) \quad -2x = (-2)(x), \quad 3y = (3)(y)\]

\[= (-2 \cdot 3)(x \cdot y) \quad \text{Commutative Property of Multiplication}\]

\[= -6xy \quad -2 \cdot 3 = -6, \quad x \cdot y = xy\]

b. Evaluate \(4ab\) if \(a = 3\) and \(b = -5\).

\[4ab = 4(3)(-5) \quad \text{Replace } a \text{ with } 3 \text{ and } b \text{ with } -5.\]

\[= [4(3)](-5) \quad \text{Associative Property of Multiplication}\]

\[= 12(-5) \quad \text{The product of 4 and 3 is positive.}\]

\[= -60 \quad \text{The product of 12 and } -5 \text{ is negative.}\]

4A. Simplify \(-3(6y)\).

4B. Simplify \(-7a(3b)\).

4C. Evaluate \(2rs\) if \(r = 5\) and \(s = -10\).

Find each product.

1. \(-3 \cdot 8\)  
2. \(5(-8)\)  
3. \(-2(11)\)  
4. \(4 \cdot 30\)  
5. \(-7(-4)\)  
6. \(-6 \cdot -6\)  
7. \(-4(-2)(-6)\)  
8. \(8(-3)(-5)\)  
9. \(-5(-9)(-12)\)

**10. FITNESS** The table shows burned Calories per minute for a 120-pound person during different activities. What is the change in the number of Calories in a 120-pound person’s body if he runs for 20 minutes?

<table>
<thead>
<tr>
<th>Activity</th>
<th>Calories per Minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet Dancing</td>
<td>6</td>
</tr>
<tr>
<td>Bicycling 14 mph</td>
<td>12</td>
</tr>
<tr>
<td>Golf, carrying clubs</td>
<td>5</td>
</tr>
<tr>
<td>Handball</td>
<td>14</td>
</tr>
<tr>
<td>Running</td>
<td>18</td>
</tr>
<tr>
<td>Skateboarding</td>
<td>6</td>
</tr>
</tbody>
</table>

**Example 3** (p. 101)  
11. **MULTIPLE CHOICE** The research submarine *Alvin*, used to locate the wreck of the *Titanic*, descended at a rate of about 100 feet per minute. Which integer describes the distance *Alvin* traveled in 5 minutes?

A. \(-500\) ft  
B. \(-100\) ft  
C. \(-20\) ft  
D. 100 ft

**Example 4** (p. 102)  
**ALGEBRA** Simplify each expression.

12. \(-4 \cdot 3x\)  
13. \(7(-3y)\)  
14. \(-8c(-3d)\)

**ALGEBRA** Evaluate each expression.

15. \(-6h\), if \(h = -20\)  
16. \(-4st\), if \(s = -9\) and \(t = 3\)
Find each product.

17. \(-3 \cdot 4\)  
18. \(-7 \cdot 6\)  
19. \(4(-8)\)  
20. \(9 \cdot (-8)\)

21. \(-12 \cdot 3\)  
22. \(14(-5)\)  
23. \(6 \cdot 19\)  
24. \(4(32)\)

25. \(-8(-11)\)  
26. \(-15(-3)\)  
27. \(-5(-4)(6)\)  
28. \(5(-13)(-2)\)

H.O.T. Problems

57. OPEN ENDED Give an example of three integers whose product is negative.

58. REASONING Calculate \((-10)(5)(18)[7 + (-7)]\) mentally. Justify your answer.

59. CHALLENGE Positive integers \(A\) and \(C\) satisfy \(A(A - C) = 23\). What is the value of \(C\)?
60. **SELECT A TOOL** During a drought, the amount of water in a pond changes by \(-9\) gallons per day due to evaporation. Which of the following tools might you use to find the number of days it takes for the amount of water in a pond to change by \(-108\) gallons of water? Justify your selection(s). Then use the tool(s) to solve the problem.

- **draw a model**
- **real objects**
- **calculator**

61. **Writing in Math** Explain how the signs of factors and products are related. Include an explanation of why the product of a positive and a negative integer must be negative.

62. An airplane descends at a rate of 200 feet per minute. Write a multiplication equation that tells the altitude of the airplane after 2 minutes.

- **A** \(-200(2) = -400\)
- **B** \(200(-2) = -400\)
- **C** \(200(2) = 400\)
- **D** \(-200(-2) = 400\)

63. **GRIDDABLE** At 8:00 P.M., a temperature of 78°F was recorded. The temperature then changed at an average rate of \(-2°F\) per hour for a 15-hour period. What was the temperature in degrees Fahrenheit at 7:00 A.M.?

### ALGEBRA
Evaluate each expression if \(a = -2\), \(b = -6\), and \(c = 14\). (Lesson 2-3)

64. \(a - c\)  
65. \(a - b\)  
66. \(c - a + b\)  
67. \(b - a + c\)

68. **SWIMMING** Lincoln High School’s swim team finished the 4 \(\times\) 100-meter freestyle relay in 5 minutes 18 seconds. Prospect High School’s swim team finished the race in 5 minutes 7 seconds. Find an integer that represents Lincoln’s finish compared to Prospect’s finish. (Lesson 2-3)

Find each sum. (Lesson 2-2)

69. \(-10 + 8 + 4\)  
70. \(-4 + (-3) + (-7)\)  
71. \(9 + (-14) + 2\)

Refer to the coordinate system. Write the ordered pair that names each point. (Lesson 1-6)

72. **E**  
73. **C**

74. **B**  
75. **F**

76. **D**  
77. **A**

### PREREQUISITE SKILL
Find each quotient.

78. \(40 \div 8\)  
79. \(90 \div 15\)  
80. \(45 \div 3\)  
81. \(91 \div 7\)
**Algebra Lab**

**Dividing Integers**

You can model division by separating algebra tiles into equal-sized groups.

**ACTIVITY**

**Use positive or negative tiles to find each quotient.**

- **a.** \(10 \div 2\)
  
  Place 10 positive tiles on the mat to represent 10.

  ![Positive Tiles](image1)

  Separate the tiles into 2 equal-sized groups.

  ![Grouping Positive Tiles](image2)

  There are 5 positive tiles in each of the 2 groups.

  So, \(10 \div 2 = 5\).

  ![Result of Division](image3)

- **b.** \(-15 \div 5\)
  
  Place 15 negative tiles on the mat to represent \(-15\).

  ![Negative Tiles](image4)

  Separate the tiles into 5 equal-sized groups.

  ![Grouping Negative Tiles](image5)

  There are 3 negative tiles in each of the 5 groups.

  So, \(-15 \div 5 = -3\).

![Result of Division](image6)

**EXERCISES**

Apply what you learned to find each quotient.

1. \(12 \div 6\)
2. \(16 \div 2\)
3. \(14 \div 7\)
4. \(-8 \div 2\)
5. \(-9 \div 3\)
6. \(-6 \div 2\)
7. \(-16 \div 4\)
8. \(-5 \div 5\)
9. \(-10 \div 2\)

**ANALYZE THE RESULTS**

For Exercises 10–12, study the quotients in Exercises 1–9.

10. When the dividend and the divisor are both positive, is the quotient positive or negative? How does this compare to the sign of a product when both factors are positive?

11. When the dividend is negative and the divisor is positive, is the quotient positive or negative? How does this compare to the sign of a product when one factor is positive and one is negative?

12. **MAKE A CONJECTURE**  
Write a rule that will help you determine the sign of the quotient of two integers.
Dividing Integers

Main Ideas
- Divide integers.
- Find the average of a set of data.

New Vocabulary
mean

Reading Math
Parts of a Division
Sentence In a division sentence, like $15 \div 5 = 3$, the number you are dividing, 15, is called the dividend. The number you are dividing by, 5, is called the divisor. The result, 3, is called the quotient.

Divide Integers You can find the quotient of two integers by using the related multiplication sentence.

$$-4 \times 3 = -12 \quad \rightarrow \quad -12 \div (-4) = 3$$
$$-2 \times 5 = -10 \quad \rightarrow \quad -10 \div (-2) = 5$$

In the division sentences $-12 \div (-4) = 3$ and $-10 \div (-2) = 5$, notice that the dividends and divisors are both negative. In both cases, the quotient is positive.

You already know that the quotient of two positive integers is positive.
$$12 \div 4 = 3 \quad \quad 10 \div 2 = 5$$

These and similar examples suggest the following rule for dividing integers with the same sign.

**KEY CONCEPT**

**Dividing Integers with the Same Sign**

**Words** The quotient of two integers with the same sign is positive.

**Examples**

$$-12 \div (-3) = 4 \quad \quad 12 \div 3 = 4$$
### Divide Integers with the Same Sign

Find each quotient.

a. \(-32 \div (-8)\)

\[-32 \div (-8) = 4\]  The quotient is positive.

b. \(\frac{75}{5}\)

\[\frac{75}{5} = 75 \div 5\]  The quotient is positive.

\[= 15\]

### Check Your Progress

1A. \(35 \div 5\)

1B. \(-\frac{39}{-3}\)

---

What is the sign of the quotient of a positive and a negative integer? Look for a pattern in the following related sentences.

**Think of this factor … to find this quotient.**

\[-4 \times (-6) = 24 \rightarrow 24 \div (-4) = -6\]

\[2 \times (-9) = -18 \rightarrow -18 \div 2 = -9\]

Notice that the signs of the dividend and divisor are different. In both cases, the quotient is negative.

These and other similar examples suggest the following rule.

**KEY CONCEPT**

<table>
<thead>
<tr>
<th>Words</th>
<th>Dividing Integers with Different Signs</th>
</tr>
</thead>
<tbody>
<tr>
<td>The quotient of two integers with different signs is negative.</td>
<td></td>
</tr>
</tbody>
</table>

**Examples**

\(-12 \div 4 = -3\)

\(12 \div (-4) = -3\)

---

### Divide Integers with Different Signs

Find each quotient.

a. \(-42 \div 3\)

\[-42 \div 3 = -14\]  The quotient is negative.

b. \(\frac{48}{-6}\)

\[\frac{48}{-6} = 48 \div (-6)\]  The quotient is negative.

\[= -8\]  Simplify.

2A. \(63 \div (-7)\)

2B. \(-\frac{110}{11}\)
EXAMPLE Evaluate Algebraic Expressions

Evaluate \( ab \div (-4) \) if \( a = -6 \) and \( b = -8 \).

\[
ab \div (-4) = \frac{-6(-8)}{-4}
\]
Replace \( a \) with \(-6\) and \( b \) with \(-8\).

\[
= \frac{48}{-4} \text{ or } -12
\]
Simplify.

3. Evaluate \( 12y \div x \) if \( x = -6 \) and \( y = -3 \).

Mean (Average) Division is used in statistics to find the average, or mean, of a set of data. To find the mean of a set of numbers, find the sum of the numbers and then divide by the number of items in the data set.

WEATHER The windchill temperatures in degrees Fahrenheit for the first six days in January were \(-2, 8, 5, -9, -12, \) and \(-2\). Find the mean temperature for the six days.

\[
\frac{-2 + 8 + 5 + (-9) + (-12) + (-2)}{6} = \frac{-12}{6}
\]
Find the sum of the set of integers.

\[
= -2
\]
Divide by the number in the set.

The mean temperature is \(-2^\circ F\).

4. GOLF Linda has scores of \(-3, -2, 1, \) and \(0\) during 4 rounds of golf. Find the mean of her golf scores.

CONCEPT SUMMARY

<table>
<thead>
<tr>
<th>Words</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adding Two Integers</strong></td>
<td>To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.</td>
</tr>
<tr>
<td>(-5 + (-4) = -9)</td>
<td>(5 + 4 = 9)</td>
</tr>
<tr>
<td>To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.</td>
<td>(-5 + 4 = -1)</td>
</tr>
<tr>
<td><strong>Subtracting Two Integers</strong></td>
<td>To subtract an integer, add its additive inverse.</td>
</tr>
<tr>
<td>(5 - 9 = 5 + (-9)) or (-4)</td>
<td>(5 - (-9) = 5 + 9) or (14)</td>
</tr>
<tr>
<td><strong>Multiplying Two Integers</strong></td>
<td>The product of two integers with the same sign is positive.</td>
</tr>
<tr>
<td>(5 \cdot 4 = 20)</td>
<td>(-5 \cdot (-4) = 20)</td>
</tr>
<tr>
<td>The product of two integers with different signs is negative.</td>
<td>(-5 \cdot 4 = -20)</td>
</tr>
<tr>
<td><strong>Dividing Two Integers</strong></td>
<td>The quotient of two integers with the same sign is positive.</td>
</tr>
<tr>
<td>(20 \div 5 = 4)</td>
<td>(-20 \div (-5) = 4)</td>
</tr>
<tr>
<td>The quotient of two integers with different signs is negative.</td>
<td>(-20 \div 5 = -4)</td>
</tr>
</tbody>
</table>
**Examples 1, 2 (p. 107)**

Find each quotient.

1. \(88 \div 8\)
2. \(-20 \div (-5)\)
3. \(-36 \div -4\)
4. \(-18 \div 6\)
5. \(\frac{70}{7}\)
6. \(-\frac{81}{9}\)

**Example 3 (p. 108)**

**ALGEBRA** Evaluate each expression.

7. \(x \div 4, \text{ if } x = -52\)
8. \(\frac{s}{t}, \text{ if } s = -45 \text{ and } t = 5\)

**Example 4 (p. 108)**

9. **WEATHER** The low temperatures for 7 days in January in degrees Fahrenheit were -2, 0, 5, -1, -4, 2, and 0. Find the average for the 7-day period.

**Exercises**

**HOMEWORK**

<table>
<thead>
<tr>
<th>HOMEWORK</th>
<th>HELP</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Exercises</td>
<td>See Examples</td>
</tr>
<tr>
<td>10–15</td>
<td>1</td>
</tr>
<tr>
<td>16–21</td>
<td>2</td>
</tr>
<tr>
<td>22–27</td>
<td>3</td>
</tr>
<tr>
<td>28, 29</td>
<td>4</td>
</tr>
</tbody>
</table>

Find each quotient.

10. \(54 \div 9\)
11. \(45 \div 5\)
12. \(-27 \div (-9)\)
13. \(-64 \div (-8)\)
14. \(-72 \div (-9)\)
15. \(-60 \div (-6)\)
16. \(-77 \div 7\)
17. \(-300 \div 6\)
18. \(480 \div (-12)\)
19. \(-150 \div 10\)
20. \(\frac{600}{-20}\)
21. \(-\frac{350}{70}\)

**ALGEBRA** Evaluate each expression.

22. \(-\frac{x}{-5}, \text{ if } x = -85\)
23. \(\frac{108}{m}, \text{ if } m = -9\)
24. \(-\frac{c}{d}, \text{ if } c = -63 \text{ and } d = -7\)
25. \(-\frac{s}{t}, \text{ if } s = 52 \text{ and } t = -4\)
26. \(xy \div (-3), \text{ if } x = 9 \text{ and } y = -7\)
27. \(ab \div 6, \text{ if } a = -12 \text{ and } b = -8\)

**28. STATISTICS** Find the mean of 4, -8, 9, -3, -7, 10, and 2.

**29. BASKETBALL** In their first five games, the Jefferson Middle School basketball team scored 46, 52, 49, 53, and 45 points. What was their average number of points per game?

**ENERGY** For Exercises 30 and 31, use the information below.

The formula \(d = \left| \frac{65 - h + \ell}{2} \right|\) can be used to find degree days, where \(h\) is the high and \(\ell\) is the low temperature.

30. If Las Vegas, Nevada, had a high of 94° and a low of 80°, find the degree days.
31. If Charleston, South Carolina, had a high of 56° and a low of 32°, find the degree days.

32. **RESEARCH** Use the Internet or another resource to find the high and low temperature for your city for a day in January. Find the degree days.

**33. SPACE** The surface temperature on Mercury at night can fall to -300°F. Use the expression \(\frac{5(F - 32)}{9}\), where \(F\) represents the temperature in degrees Fahrenheit, to find the temperature on Mercury in degrees Celsius. Round to the nearest tenth.

---

**Lesson 2-5 Dividing Integers** 109
34. OPEN ENDED Write an equation with three integers that illustrates dividing integers with different signs.

35. CHALLENGE Find values for \( x, y, \) and \( z \) so that all of the following statements are true.

- \( y > x, \) \( z < y, \) and \( x < 0 \)
- \( z \div 2 \) and \( z \div 3 \) are integers.
- \( x \div z = -z \)
- \( x \div y = z \)

36. CHALLENGE Addition and multiplication are said to be closed for whole numbers, but subtraction and division are not. That is, when you add or multiply any two whole numbers, the result is a whole number. Which operations are closed for integers?

37. Writing in Math Use the information about dividing integers on pages 106–107 to explain how dividing integers is related to multiplying integers. Illustrate your answer with two related multiplication and division sentences.

38. The table shows the sales of a computer chip manufacturer in two recent years. What is the average change in sales per year?

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>$115</td>
</tr>
<tr>
<td>2000</td>
<td>$128</td>
</tr>
</tbody>
</table>

\[ \text{Average change} = \frac{128 - 115}{2} = \frac{13}{2} = 6.5 \text{ million} \]

39. Pedro has quiz scores of 8, 7, 8, and 9. What is the lowest score he can get on the remaining quiz to have a final average (mean) score of at least 8?

A $7 G $8 H $9 J $10

40. \(-8 - (-25)\)

41. \(75 - 114\)

42. \(2ab \cdot (-2)\)

43. \((-10c)(5d)\)

44. ANIMALS The height of an adult giraffe is 3 times the height of a newborn giraffe. Given \( n \), the height of a newborn giraffe, write an equation that can be used to find \( a \), the height of an adult giraffe. (Lesson 1-5)

45. PATTERNS Find the next two numbers in the pattern 5, 4, 2, -1, ….

A $13 million C $2.6 million

B $2.6 million D $13 million

46. (1, 5)

47. (6, 2)

48. (4, 5)

49. (0, 3)

Find each difference or product. (Lessons 2-3 and 2-4)

\( -8 - (-25) \quad 75 - 114 \quad 2ab \cdot (-2) \quad (-10c)(5d) \)

44. ANIMALS The height of an adult giraffe is 3 times the height of a newborn giraffe. Given \( n \), the height of a newborn giraffe, write an equation that can be used to find \( a \), the height of an adult giraffe. (Lesson 1-5)

45. PATTERNS Find the next two numbers in the pattern 5, 4, 2, -1, …. (Lesson 1-1)
The Coordinate System

Main Ideas
- Graph points on a coordinate plane.
- Graph algebraic relationships.

New Vocabulary
quadrants

A GPS, or Global Positioning System, can be used to find a location anywhere on Earth by identifying its latitude and longitude. Several cities are shown on the map below. For example, Brisbane, Australia, is located at approximately 30°S, 150°E.

Graph Points
Latitude and longitude are a kind of coordinate system. The coordinate system, or coordinate plane you used in Lesson 1-6 can be extended to include points below and to the left of the origin.

Recall that a point graphed on the coordinate system has an \( x \)-coordinate and a \( y \)-coordinate. The dot at the ordered pair \((-4, -2)\) is the graph of point \( P \).
EXAMPLE  Write Ordered Pairs

Write the ordered pair that names each point.

a. \(A\)

\(x\)-coordinate is \(-3\).
\(y\)-coordinate is \(2\).
The ordered pair is \((-3, 2)\).

b. \(B\)

\(x\)-coordinate is \(4\).
\(y\)-coordinate is \(-2\).
The ordered pair is \((4, -2)\).

The coordinates are (negative, positive).
The coordinates are (positive, positive).
The coordinates are (negative, negative).
The coordinates are (positive, negative).

Ordered Pairs
Notice that the axes in an ordered pair \((x, y)\) are listed in alphabetical order.

EXAMPLE  Graph Points and Name the Quadrant

Graph and label each point on a coordinate plane. Name the quadrant in which each point lies.

a. \(E(2, 4)\)

Start at the origin. Move 2 units right.
Then move 4 units up and draw a dot.
Point \(E(2, 4)\) is in Quadrant I.

b. \(F(-3, -2)\)

Start at the origin. Move 3 units left.
Then move 2 units down and draw a dot.
Point \(F(-3, -2)\) is in Quadrant III.

c. \(G(4, 0)\)

Start at the origin. Move 4 units right. Since the \(y\)-coordinate is 0, the point lies on the \(x\)-axis. Point \(G(4, 0)\) is not in any quadrant.

Check Your Progress

1A. \(C\) \hspace{1cm} 1B. \(D\)

2A. \(H(4, -3)\) \hspace{1cm} 2B. \(J(0, -2)\) \hspace{1cm} 2C. \(I(-1, 4)\)

The \(x\)-axis and the \(y\)-axis separate the coordinate plane into four regions, called quadrants. The axes and points on the axes are not located in any of the quadrants. The quadrants are named I, II, III, and IV.
**Graph Algebraic Relationships** You can use a coordinate graph to show relationships between two numbers.

### Example 1
**Graph an Algebraic Relationship**

The sum of two numbers is 5. If \( x \) represents the first number and \( y \) represents the second number, make a table of possible values for \( x \) and \( y \). Graph the ordered pairs and describe the graph.

First, make a table. Choose values for \( x \) and \( y \) that have a sum of 5. Then graph the ordered pairs on a coordinate plane.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>(-1)</td>
<td>6</td>
<td>((-1, 6))</td>
</tr>
<tr>
<td>(-2)</td>
<td>7</td>
<td>((-2, 7))</td>
</tr>
</tbody>
</table>

The points on the graph are in a line that slants downward to the right. The line crosses the \( y \)-axis at \( y = 5 \).

### Check Your Progress

3. The difference of two numbers is 4. If \( x \) represents the first number and \( y \) represents the second number, make a table of possible values for \( x \) and \( y \). Graph the ordered pairs and describe the graph.

### Example 1 (p.112)

Name the ordered pair for each point graphed at the right.

1. \( A \)  
2. \( C \)  
3. \( G \)  
4. \( K \)

### Example 2 (p.112)

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

5. \( J(3, -4) \)  
6. \( K(-2, 2) \)  
7. \( L(0, 4) \)  
8. \( M(-1, -2) \)

9. **GEOMETRY** Graph points \( A(-4, 3), B(1, 3), C(1, 2), \) and \( D(-4, 2) \) on a coordinate plane and connect them to form a rectangle. Name the quadrant in which each point is located.

### Example 3 (p.113)

10. **ALGEBRA** Make a table of values and graph six ordered integer pairs where \( x + y = 3 \). Describe the graph.
Name the ordered pair for each point graphed at the right.

15. V  16. H
17. U  18. W
19. A  20. T

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

25. S(2, -5)  26. F(-4, 0)  27. E(0, 3)  28. K(-5, 1)
29. G(5, 0)  30. C(6, -1)  31. D(0, 0)  32. R(-3, 5)

ALGEBRA Make a table of values and graph six sets of ordered integer pairs for each equation. Describe the graph.

33. x + y = 4  34. x - y = -2  35. y = 2x
36. y = -2x  37. y = x + 2  38. y = x - 1

Graph each point. Then connect the points in alphabetical order and identify the figure.

39. A(0, 6), B(4, -6), C(-6, 2), D(6, 2), E(-4, -6), F(0, 6)
40. A(5, 8), B(1, 13), C(5, 18), D(9, 13), E(5, 8), F(5, 6), G(3, 7), H(3, 5), I(7, 7), J(7, 5), K(5, 6), L(5, 3), M(3, 4), N(3, 2), P(7, 4), Q(7, 2), R(5, 3), S(5, 1)

GEOMETRY On a coordinate plane, draw triangle ABC with vertices at A(3, 1), B(4, 2), and C(2, 4). Then graph and describe each new triangle formed in Exercises 41–44.

41. Multiply each coordinate of the vertices in triangle ABC by 2.
42. Multiply each coordinate of the vertices in triangle ABC by -1.
43. Add 2 to each coordinate of the vertices in triangle ABC.
44. Subtract 4 from each coordinate of the vertices in triangle ABC.

RESEARCH Find a map of your school and draw a coordinate grid on the map with the library as the center. Locate the cafeteria, principal’s office, your math classroom, gym, counselor’s office, and the main entrance on your grid. Write the coordinates of these places. How can you use these points to help visitors find their way around your school?

Graph and label each point on a coordinate plane.

46. A(-6.5, 3)  47. B(-2, -5.75)  48. C(4.1, -1)  49. D(-3.4, 1.5)

ALGEBRA Graph eight ordered integer pairs where |x| > 3. Describe the graph.

50. ALGEBRA Graph all ordered integer pairs that satisfy the condition |x| < 4 and |y| < 3.
52. **OPEN ENDED** Name two ordered pairs whose graphs are not located in one of the four quadrants.

53. **FIND THE ERROR** Keisha says that if you interchange the coordinates of any point in Quadrant I, the new point would be in Quadrant I. Jason says the new point would be in Quadrant III. Who is correct? Explain your reasoning.

**CHALLENGE** If the graph of \(A(x, y)\) satisfies the given condition, name the quadrant in which point \(A\) is located.

54. \(x > 0, \ y > 0\)  
55. \(x < 0, \ y < 0\)  
56. \(x < 0, \ y > 0\)

57. **NUMBER SENSE** Graph eight sets of integer coordinates that satisfy \(|x| + |y| > 3\). Describe the location of the points.

58. **Writing in Math** Use the information on page 111 to explain how a coordinate plane is used to locate places on Earth. Include an explanation of how coordinates can describe a location and how latitude and longitude are related to the \(x\)- and \(y\)-axes on a coordinate plane.

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For Exercises 59 and 60, refer to the graph at the right.

59. On the coordinate plane, what are the coordinates of the point that shows the location of the library?
   - A (4, -2)  
   - B (−2, −4)  
   - C (4, 2)  
   - D (−4, −2)

60. On the coordinate plane, what location has coordinates \((5, -2)\)?
   - F park  
   - G school  
   - H library  
   - J store

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**Spiral Review**

Find each quotient. (Lesson 2-5)

61. \(-24 \div (-8)\)  
62. \(105 \div (-5)\)  
63. \(-400 \div (-50)\)

**ALGEBRA** Evaluate each expression if \(f = -9, \ g = -6, \) and \(h = 8\). (Lesson 2-4)

64. \(-5fg\)  
65. \(2gh\)  
66. \(-10fh\)

67. **WEATHER** In the newspaper, Ruben read that the low temperature for the day was expected to be \(-5^\circ F\) and the high temperature was expected to be \(8^\circ F\). What was the difference in the expected high and low temperatures? (Lesson 2-3)

**ALGEBRA** Simplify each expression. (Lesson 1-4)

68. \((a + 8) + 6\)  
69. \(4(6h)\)  
70. \((n \cdot 7) \cdot 8\)

71. \((b \cdot 9) \cdot 5\)  
72. \((16 + 3y) + y\)  
73. \(0(4z)\)
Key Concepts

Integers and Absolute Value (Lesson 2-1)
- Numbers on a number line increase as you move from left to right.
- The absolute value of a number is the distance the number is from zero on the number line.

Adding and Subtracting Integers (Lessons 2-2 and 2-3)
- To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.
- To add integers with different signs subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.
- To subtract an integer, add its additive inverse.

Multiplying and Dividing Integers (Lessons 2-4 and 2-5)
- The product or quotient of two integers with the same sign is positive.
- The product or quotient of two integers with different signs is negative.

The Coordinate Plane (Lesson 2-6)
- The x-axis and the y-axis separate the coordinate plane into four quadrants.
- The axes and points on the axes are not located in any of the quadrants.

Key Vocabulary

absolute value (p. 80)
additive inverse (p. 88)
coordinate (p. 79)
inequality (p. 79)
integers (p. 78)
mean (p. 108)
negative number (p. 78)
opposites (p. 88)
quadrants (p. 112)

Vocabulary Check

Complete each sentence with the correct term. Choose from the list above.
1. A(n) _______ is a number less than zero.
2. The number that corresponds to a point on the number line is called the _______.
3. An integer and its opposite are also called _______ of each other.
4. The four regions separated by the axes on a coordinate plane are called _______.
5. The set of _______ includes positive whole numbers, their opposites, and zero.
6. The _______ of a number is the distance the number is from zero on the number line.
7. A(n) _______ is a mathematical sentence containing < or >.
8. To find the _______ of a set of numbers, find the sum of the numbers and then divide by the number of items in the data set.
9. Two numbers with the same absolute values but different signs are _______.

Download Vocabulary Review from pre-alg.com
**Lesson-by-Lesson Review**

### 2-1 Integers and Absolute Value (pp. 78-83)

**Example 1** Replace each with <, >, or = to make a true sentence.

10. \(8 \; \bullet \; -8\)  
11. \(-3 \; \bullet \; -3\)  
12. \(-2 \; \bullet \; 0\)  
13. \(-12 \; \bullet \; -21\)

**Evaluate each expression.**

14. \(|-32|\)  
15. \(|25|\)  
16. \(|-15|\)  
17. \(|-8| + |-14|\)

**BASEBALL CARDS** Jamal traded away 7 shortstop cards for 5 outfielder cards. Find an integer that represents the change in the number of cards Jamal had after the trade.

### 2-2 Adding Integers (pp. 86-90)

**Example 3** Find \(-3 + (-4)\).

\[-3 + (-4) = -7\] The sum is negative.

**Example 4** Find \(5 + (-2)\).

\[5 + (-2) = 3\] The sum is positive.

**Find each sum.**

19. \(-6 + (-3)\)  
20. \(-4 + (-1)\)  
21. \(-2 + 7\)  
22. \(4 + (-8)\)  
23. \(4 + 7 + (-3)\)  
24. \(-9 + 6 + (-8)\)  
25. **GOLF** A golfer’s scores for the last five weeks are \(-5, +7, -2, -4,\) and \(+5\). What is the sum of his scores?

### 2-3 Subtracting Integers (pp. 93-97)

**Example 5** Find \(-5 - 2\).

\[-5 - 2 = -5 + (-2)\] To subtract 2, add \(-2\).

\[= -7\]

**Example 6** Find \(8 - (-4)\).

\[8 - (-4) = 8 + 4\] To subtract \(-4,\) add 4.

\[= 12\]

**Find each difference.**

26. \(4 - 9\)  
27. \(-3 - 5\)  
28. \(7 - (-2)\)  
29. \(-1 - (-6)\)  
30. \(-7 - 8\)  
31. \(6 - 10\)  
32. **ELEVATORS** The postal carrier entered the elevator on floor 15. She rode down 6 floors. Then she rode up 10 floors and got off. What floor was she on when she left the elevator?
2–4  Multiply ing Integers  (pp. 100–104)

Find each product.
33. \(-9(5)\)  34. \(11(-6)\)
35. \(-4(-7)\)  36. \(-3(-16)\)

37. SNOWBOARDING For each trick he completes incorrectly in the half pipe event, Kurt receives \(-3\) points. If Kurt incorrectly completes five tricks, what is his score?

Example 7  Find \(6(-4)\).
\[6(-4) = -24\]  The factors have different signs, so the product is negative.

Example 8  Find \(-8(-2)\).
\[-8(-2) = 16\]  The factors have the same sign, so the product is positive.

2–5  Dividing Integers  (pp. 106–110)

Find each quotient.
38. \(-14 \div (-2)\)  39. \(-52 \div (-4)\)
40. \(-36 \div 9\)  41. \(88 \div (-4)\)

42. RACING The number of seconds Elena is behind the leader for the first five legs of the bicycle race is shown. What is her average time behind the leader? \(+32\) s, \(+5\) s, \(+10\) s, \(+8\) s, \(+12\) s

Example 9  Find \(-30 \div (-5)\).
\[-30 \div (-5) = 6\]  The dividend and divisor have the same sign, so the quotient is positive.

Example 10  Find \(27 \div (-3)\).
\[27 \div (-3) = -9\]  The dividend and divisor have different signs, so the quotient is negative.

2–6  The Coordinate System  (pp. 111–115)

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.
43. \(A(4, 3)\)  44. \(J(-2, -5)\)
45. \(K(-1, 3)\)  46. \(R(3, 0)\)

47. GAMES The coordinate plane represents a board game. Name the quadrant in which each player’s game piece is located.

Example 11  Graph and label \(F(5, -3)\) on a coordinate plane. Name the quadrant in which the point is located.
Point \(F(5, -3)\) is in Quadrant IV.
Write two inequalities using the numbers in each sentence. Use the symbols < and >.

1. \(-5\) is less than \(2\).
2. \(12\) is greater than \(-15\).

3. **MULTIPLE CHOICE** A scuba diver records her depth in the lake every minute. Choose the group of depths that is listed in order from least to greatest.
   - A \(-13\) ft, \(-12\) ft, \(-9\) ft, \(-3\) ft, \(-1\) ft, \(-5\) ft
   - B \(-5\) ft, \(-3\) ft, \(-1\) ft, \(-9\) ft, \(-12\) ft, \(-13\) ft
   - C \(-12\) ft, \(-13\) ft, \(-3\) ft, \(-1\) ft, \(-9\) ft, \(-5\) ft
   - D \(-13\) ft, \(-12\) ft, \(-9\) ft, \(-5\) ft, \(-3\) ft, \(-1\) ft

4. **FOOTBALL** During the first play of the game, the Brownville Tigers football team lost seven yards. On each of the next two plays, an additional four yards were lost. Express the total yards lost at the end of the first three plays as an integer.

Find each sum or difference.

5. \(-4 + (-8)\)
6. \(-9 + 15\)
7. \(12 + (-15)\)
8. \(14 + (-7) + (-11)\)
9. \(4 - 13\)
10. \(8 - (-6)\)
11. \(-6 - (-10)\)
12. \(-14 - (-7)\)

13. **STOCK MARKET** On Thursday, a company’s stock closed at $67.24. On Friday, it closed at $64.27. What was the change in the closing price?

Find each product or quotient.

14. \(6(-8)\)
15. \(-9(8)\)
16. \(-7(-5)\)
17. \(2(-4)(11)\)
18. \(54 \div (-9)\)
19. \(-64 \div (-4)\)
20. \(-250 \div 25\)
21. \(-144 \div (-6)\)

22. **SWIMMING POOL** The water in a swimming pool drains at a rate of 24 gallons per minute. Describe the change in the amount of water in the swimming pool after 1 hour.

23. **WEATHER** The table shows the low temperatures during one week in Anchorage, Alaska. Find the average low temperature for the week.

<table>
<thead>
<tr>
<th>Day</th>
<th>S</th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>T</th>
<th>F</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>-12</td>
<td>3</td>
<td>-7</td>
<td>0</td>
<td>-4</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

**ALGEBRA** Evaluate each expression if \(a = -5\), \(b = 3\), and \(c = -10\).

24. \(ab - c\)
25. \(c \div a\)
26. \(\frac{bc}{a} - 6\)
27. \(4c + |a|\)

28. **MULTIPLE CHOICE** A vertex of a triangle is a point where two sides of the triangle meet. Which ordered pair is not a vertex of \(\triangle ABC\)?
   - F \((-1, 1)\)
   - G \((2, -3)\)
   - H \((1, 2)\)
   - J \((-1, -1)\)

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

29. \(D(-2, 4)\)
30. \(E(3, -4)\)
31. \(F(-1, -3)\)
32. \(G(3, 2)\)

33. **MULTIPLE CHOICE** Suppose Elan’s home represents the origin on a coordinate plane. If Elan leaves his home and walks two miles west and then four miles north, what is his current location as an ordered pair?
   - A \((-2, 4)\)
   - B \((2, 4)\)
   - C \((-2, -4)\)
   - D \((4, -2)\)
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A scuba diver descends at a rate of 40 feet per minute. Which equation shows how far the scuba diver moves in 2 minutes?
   A \(-40(-2) = 80\)  C \(40(2) = 80\)
   B \(-40(2) = -80\)  D \(40(-2) = -80\)

2. On Wednesday, the low temperature in Fairbanks, Alaska was \(-6^\circ F\), and the high temperature was \(14^\circ F\). How much warmer was the high temperature than the low temperature?
   F \(-20^\circ F\)  H \(8^\circ F\)
   G \(-8^\circ F\)  J \(20^\circ F\)

3. **GRIDDABLE** Michael had \$45 in his savings account at the beginning of the week. He made a withdrawal of \$22 to buy a video game on Tuesday, and he made a deposit of \$25 on Friday when he received some money for his birthday. How much money in dollars did Michael have in the account at the end of the week if he made no other withdrawals or deposits?

4. Tyrone’s long distance phone bills were \$21.35, \$11.14, \$22.82, and \$33.05 over the past four months. He estimated that the phone bill would cost \$80 over these four months. Which statement best describes how reasonable his estimate is?
   A Less than the actual amount because he rounded to the nearest \$10
   B Less than the actual amount because he rounded to the nearest \$100
   C More than the actual amount because he rounded to the nearest \$10
   D More than the actual amount because he rounded to the nearest \$100

5. What are the coordinates of the center of the circle below?
   F \((-3, 2)\)  G \((-2, 3)\)
   H \((3, -2)\)  J \((2, -3)\)

6. **GRIDDABLE** What is the twelfth term of the pattern given by the expression below where \(n\) is the term number?
   \[3(n - 5)\]

7. Last week, Traci wrote checks for \$32, \$58, and \$14. She also made two deposits totaling \$189. What other information is needed in order to find the current balance in Traci’s account?
   A The amount of each deposit.
   B Traci’s balance last week.
   C To whom Traci wrote checks last week.
   D Traci’s deposit when she opened the account.

8. Of the six books in a mystery series, four have 200 pages and two have 300 pages. Which expression represents the total number of pages in the series?
   F \(200 + 300\)  H \(4(200) + 2(300)\)
   G \(6(200 + 300)\)  J \(8(200 + 300)\)

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9. A pattern of equations is shown below.

1% of 2,000 = 20
2% of 1,000 = 20
4% of 500 = 20
8% of 250 = 20

Which statement best describes this pattern?
A. When the percent is doubled and the other number is doubled, the answer is 20.
B. When the percent is doubled and the other number is halved, the answer is 20.
C. When the percent is increased by 2 and the other number remains the same, the answer is 20.
D. When the percent remains the same and the other number is increased by 2, the answer is 20.

10. Tonya wants to order a roast beef sandwich, a medium order of fries, and a medium drink. How much money will she save by ordering a Daily Special #2?

F. $1.22
G. $0.84
H. $0.78
J. $0.38

11. Before the last game of the season, Amy had scored a total of 58 goals. She scored 4 goals in the final game, making her season average 3.1 goals per game. To find the total number of games that Amy played, first find the sum of 58 and 4, and then—
A. add the sum to 3.1.
B. subtract 3.1 from 58.
C. multiply the sum by 3.1.
D. divide the sum by 3.1.

Question 12: When answering open-ended items on standardized tests, follow these steps:
1. Read the item carefully.
2. Show all of your work. You may receive points for items that are only partially correct.
3. Check your work.

Record your answers on a sheet of paper.
Show your work.

12. On graph paper, graph the points A(4, 2), B(−3, 7), and C(−3, 2). Connect the points to form a triangle.

a. Add 6 to the x-coordinate of each coordinate pair. Graph and connect the new points to form a new figure. Is the new figure the same size and shape as the original triangle? Describe how the size, shape, and position of the new triangle relate to the size, shape, and position of the original triangle.

b. If you add −6 to each original x-coordinate, and graph and connect the new points to create a new figure, how will the position of the new figure relate to that of the original one?